Signal Analysis & Communication ECE355

Review of Midterm 2 Content.

Lecture 26

15-11-2023

REVIEW

- Response of LTI System to Complex Exponentials

$$x_{(t)} = e^{st} \longrightarrow \boxed{h(t)} \longrightarrow y_{(t)} = H_{(s)}\, e^{st}$$

where $H_{(s)} = \int_{-\infty}^{\infty} h_{(t)}\, e^{-st}\, dt$

OR

$$x_{(t)} = e^{j\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow y_{(t)} = H_{(j\omega_0)}\, e^{j\omega_0 t}$$

where $H(j\omega_0) = \int_{-\infty}^{\infty} h_{(t)}\, e^{-j\omega_0 t}\, dt$

Also,

$$x_{(t)} = \sum_k a_k e^{j\omega_k t} \longrightarrow \boxed{h(t)} \longrightarrow y_{(t)} = \sum_k a_k H_{(j\omega_k)}\, e^{j\omega_k t}$$

- Similarly for DT case.

Periodic Signals in CT

- "Almost all" periodic signals can be expressed as a sum of harmonically related complex sinusoids.

CT

Periodic

↕

Aperiodic

Discrete

$$x_{(t)} = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0} \qquad \text{SYNTHESIS}$$

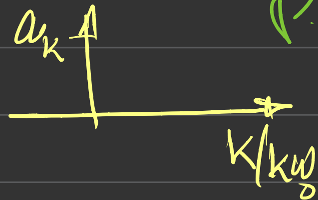
where

$$a_k = \frac{1}{T_0} \int_{T_0} x_{(t)}\, e^{-jk\omega_0 t}\, dt. \qquad \text{ANALYSIS}$$

$\boxed{\text{CTFS.}}$

$x_{(t)}$: time domain representation, a_k: freq. domain representation how the sig. varies with time.

which freq. components are present in the sig. & with what amplitude. (Freq. content of the sig.)



CTFS – Properties – helpful (Formula sheet).

CTFS – Convergence I. $\int_{T_0} |x_{(t)}|^2 dt < \infty$

II. a) Finite discontinuities over T_0

b) Finite no. of maxima/minima over T_0.

c) $\int_{T_0} |x_{(t)}| \, dt < \infty$

Aperiodic Signals in CT

- Ref. to Periodic sig. in CT: $T_0 \to \infty$, $k\omega_0 \to \omega$

$\Sigma \to \int$

CT Aperiodic

$$x_{(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_{(j\omega)} e^{j\omega t} d\omega$$

SYNTHESIS

where

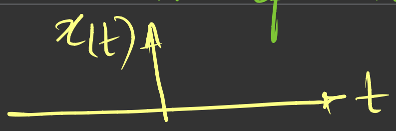
$$X_{(j\omega)} = \int_{-\infty}^{\infty} x_{(t)} e^{-j\omega t} dt$$

ANALYSIS.

Aperiodic Continuous

$\boxed{\text{CTFT}}$

$x_{(t)}$: time-domain representation, $X_{(j\omega)}$: freq.-domain representation



- Well Known CTFT Pairs. (Formula sheet).

- CTFT of Periodic Signals (unified context).

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \underbrace{\delta(\omega - k\omega_0)}_{\text{Impulses at } \omega = k\omega_0,\ \text{area of impulses} \propto a_k}$$

Impulses at $\omega = k\omega_0$
area of impulses $\propto a_k$.

- Properties of CTFT - Useful (Formula Sheet).

- Remember Duality:

$$\boxed{\begin{array}{lll} \text{If} & x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega) & (= g(\omega)) \\ \text{then} & g(t) \longleftrightarrow 2\pi x(-\omega) & \end{array}}$$

- Also, remember multiplication & Convolution properties (Formula Sheet).

- Convergence of CTFT - Similar to CTFS.

Periodic Signals in DT

- Remember DT signals are periodic in freq. with period 2π:

$$e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi)n} \quad \text{OR} \quad e^{j\omega n} = e^{j(\omega + 2\pi)n}.$$

$\Rightarrow$ DT Periodic: $e^{jk\frac{2\pi}{N}n} = e^{j(k+N)\frac{2\pi}{N}n}$ is periodic in freq.

with period N. $\omega_0 = \frac{2\pi}{N}$.

DT Periodic $\leftrightarrow$ Periodic Discrete

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \qquad \text{SYNTHESIS}$$

where

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} \qquad \text{ANALYSIS.}$$

$\boxed{\text{DTFS}}$

- Remember a_k is periodic with period N.

$x[n]$: time-domain representation, a_k : freq.-domain representation.

- Remember the FACT:

$$\boxed{\sum_{n=\langle N \rangle} e^{jk\frac{2\pi}{N}n} = \begin{cases} N, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise.} \end{cases}}$$

- Properties of DTFS - useful (Formula Sheet).

Aperiodic Signals in DT

- Remember DT sig. are periodic in freq. with period 2π.
- Ref. to periodic sig. in DT: $N \to \infty$, $k\omega_0 \to \omega$

DT Aperiodic

$\updownarrow$

Periodic Continuous

$$x_{(t)} = \frac{1}{2\pi} \int_{2\pi} X_{(e^{j\omega})} e^{j\omega n} d\omega \quad \text{SYNTHESIS.}$$

where

$$X_{(e^{j\omega})} = \sum_{n=-\infty}^{\infty} x_{[n]} e^{-j\omega n} \quad \text{ANALYSIS.}$$

$\boxed{\text{DTFT}}$

- Convergence Conditions.

$$\sum_{n=-\infty}^{\infty} |x_{[n]}| < \infty$$

OR

$$\sum_{n=-\infty}^{\infty} |x_{[n]}|^2 < \infty$$

- DTFT of Periodic Signals

$$X_{(e^{j\omega})} = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta_{(\omega - k\omega_0)}.$$

KNOW

- Lowpass Filter / Highpass Filter / Bandpass Filter

LCCDE

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y_{(t)} = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x_{(t)}$$

$$H_{(s)} = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} \xleftrightarrow{\mathcal{F}^{-1}} h_{(t)}$$

Use Partial Fraction Expansion

Also, $$Y_{(s)} = H_{(s)} X_{(s)} \xleftrightarrow{\mathcal{F}^{-1}} y_{(t)}$$

Make use of well known pairs.

Remember,

$$x_{(t)} * \delta_{(t)} = x_{(t)}$$

$$x_{(t)} * \delta_{(t-\tau)} = x_{(t-\tau)}$$

$$X_{(j\omega)} * \delta_{(j\omega)} = X_{(j\omega)}$$

$$X_{(j\omega)} * \delta_{(j(\omega-\omega_0))} = X_{(j(\omega-\omega_0))}$$